Formelblad matematik 5

Algebra

Regler

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Andragradsekvationer

$$x^2 + px + q = 0 \qquad ax^2 + bx + c = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \qquad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Binomialsatsen

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{n} b^n$$

Aritmetik

Prefix

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hekto	deci	centi	milli	mikro	nano	piko
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Potenser

$$a^x a^y = a^{x+y} \qquad \frac{a^x}{a^y} = a^{x-y} \qquad (a^x)^y = a^{xy} \qquad a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^0 = 1$$

Logaritmer

$$y = 10^x \Leftrightarrow x = \lg y \qquad y = \mathrm{e}^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy \qquad \lg x - \lg y = \lg \frac{x}{y} \qquad \lg x^p = p \cdot \lg x$$

Absolutbelopp

$$|a| = \begin{cases} a & \text{om } a \geq 0 \\ -a & \text{om } a < 0 \end{cases}$$

Funktioner och samband

Räta linjen

$y = kx + m$ $\qquad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$ax + by + c = 0$, där inte både a och b är noll

Andragradsfunktioner

$y = ax^2 + bx + c \qquad a \neq 0$

Potensfunktioner

$y = C \cdot x^a$

Exponentialfunktioner

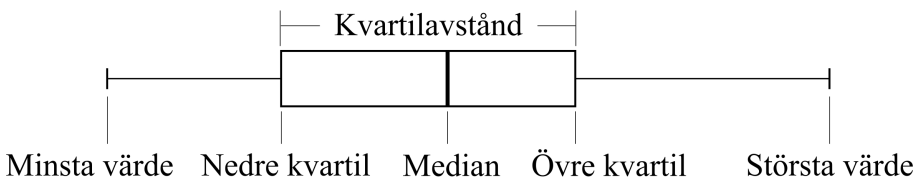
$y = C \cdot a^x \qquad a > 0$ och $a \neq 1$

Statistik och sannolikhet

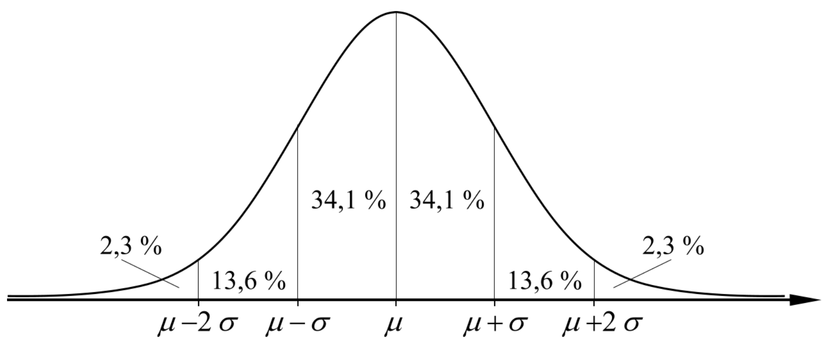
Standardavvikelse för ett stickprov

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n - 1}}$$

Lådagram



Normalfördelning



Täthetsfunktion för normalfördelning

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential- och integralkalkyl

Derivatans definition

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivator

Funktion	Derivata
x^n där n är ett reellt tal	nx^{n-1}
a^x $(a > 0)$	$a^x \ln a$
$\ln x$ $(x > 0)$	$\frac{1}{x}$
e^x	e^x
e^{kx}	$k \cdot e^{kx}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$ $(g(x) \neq 0)$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Kedjeregeln

Om $y = f(z)$ och $z = g(x)$ är två deriverbara funktioner så gäller för $y = f(g(x))$ att

$$y' = f'(g(x)) \cdot g'(x) \text{ eller } \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

Primitiva funktioner

Funktion	Primitiva funktioner
k	$kx + C$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln x + C \quad (x > 0)$
e^x	$e^x + C$
e^{kx}	$\dfrac{e^{kx}}{k} + C$
$a^x \quad (a > 0,\ a \neq 1)$	$\dfrac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Komplexa tal

Representation $z = x + iy = re^{iv} = r(\cos v + i\sin v)$ där $i^2 = -1$

Argument $\arg z = v \qquad \tan v = \dfrac{y}{x}$

Absolutbelopp $|z| = r = \sqrt{x^2 + y^2}$

Konjugat Om $z = x + iy$ så $\bar{z} = x - iy$

Räknelagar

$$z_1 z_2 = r_1 r_2(\cos(v_1 + v_2) + i\sin(v_1 + v_2))$$

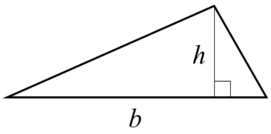
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(v_1 - v_2) + i\sin(v_1 - v_2))$$

de Moivres formel $z^n = (r(\cos v + i\sin v))^n = r^n(\cos nv + i\sin nv)$

Geometri

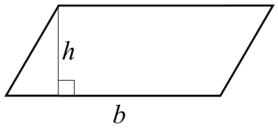
Triangel

$$A = \frac{bh}{2}$$



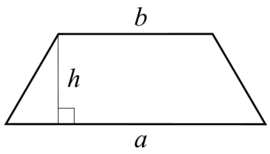
Parallellogram

$$A = bh$$



Parallelltrapets

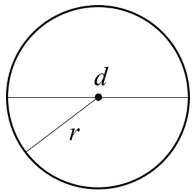
$$A = \frac{h(a+b)}{2}$$



Cirkel

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

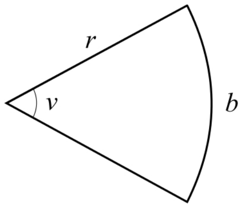
$$O = 2\pi r = \pi d$$



Cirkelsektor

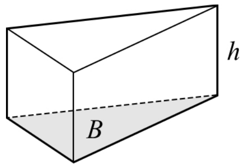
$$b = \frac{v}{360°} \cdot 2\pi r$$

$$A = \frac{v}{360°} \cdot \pi r^2 = \frac{br}{2}$$



Prisma

$$V = Bh$$

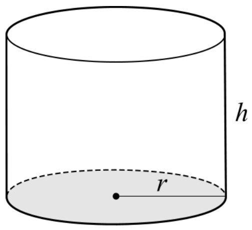


Cylinder

$$V = \pi r^2 h$$

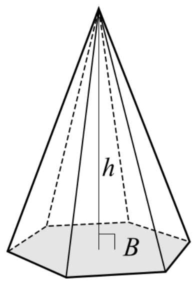
Mantelarea

$$A = 2\pi rh$$



Pyramid

$$V = \frac{Bh}{3}$$

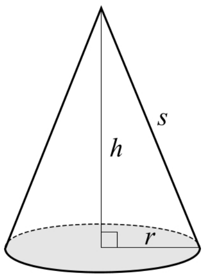


Kon

$$V = \frac{\pi r^2 h}{3}$$

Mantelarea

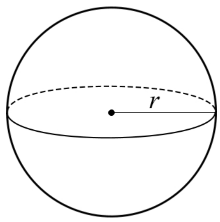
$$A = \pi rs$$



Klot

$$V = \frac{4\pi r^3}{3}$$

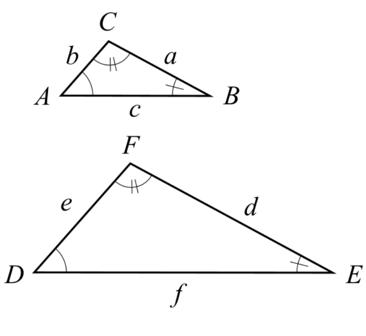
$$A = 4\pi r^2$$



Likformighet

Trianglarna *ABC* och *DEF* är likformiga.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



Skala

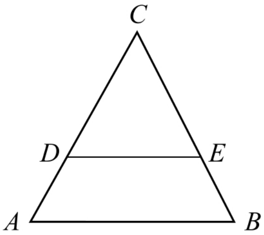
Areaskalan = (Längdskalan)^2
Volymskalan = (Längdskalan)^3

Topptriangel- och transversalsatsen

Om DE är parallell med AB gäller

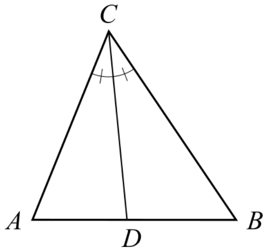
$$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC} \text{ och}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$



Bisektrissatsen

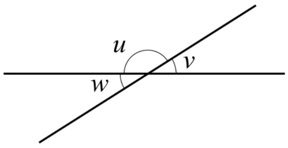
$$\frac{AD}{BD} = \frac{AC}{BC}$$



Vinklar

$u + v = 180°$ Sidovinklar

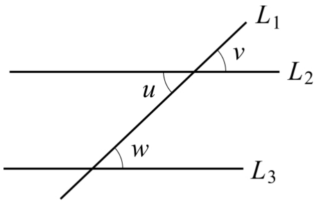
$w = v$ Vertikalvinklar



L_1 skär två parallella linjer L_2 och L_3

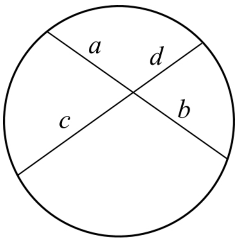
$v = w$ Likbelägna vinklar

$u = w$ Alternatvinklar



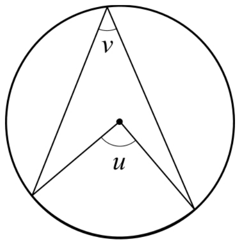
Kordasatsen

$ab = cd$



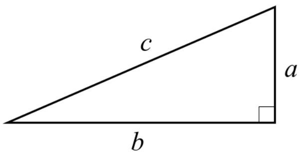
Randvinkelsatsen

$u = 2v$



Pythagoras sats

$a^2 + b^2 = c^2$



Avståndsformeln

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Mittpunktsformeln

$$x_m = \frac{x_1 + x_2}{2} \text{ och } y_m = \frac{y_1 + y_2}{2}$$

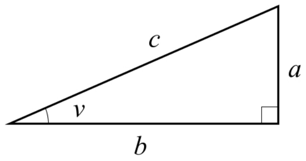
Trigonometri

Definitioner

$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

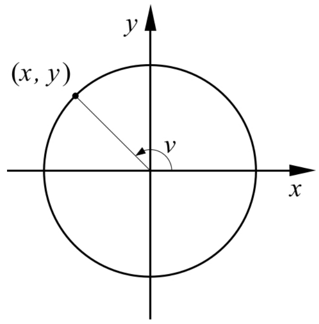


Enhetscirkeln

$$\sin v = y$$

$$\cos v = x$$

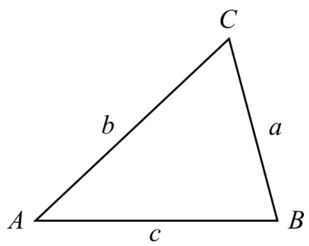
$$\tan v = \frac{y}{x}$$



Sinussatsen $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosinussatsen $$a^2 = b^2 + c^2 - 2bc\cos A$$

Areasatsen $$T = \frac{ab\sin C}{2}$$



Trigonometriska formler

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\sin 2v = 2\sin v\cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2\cos^2 v - 1 & (2) \\ 1 - 2\sin^2 v & (3) \end{cases}$$

$$a\sin x + b\cos x = c\sin(x+v) \text{ där } c = \sqrt{a^2+b^2} \text{ och } \tan v = \frac{b}{a}$$

Cirkelns ekvation $$(x-a)^2 + (y-b)^2 = r^2$$

Exakta värden

Vinkel v (grader) (radianer)	$0°$ 0	$30°$ $\frac{\pi}{6}$	$45°$ $\frac{\pi}{4}$	$60°$ $\frac{\pi}{3}$	$90°$ $\frac{\pi}{2}$	$120°$ $\frac{2\pi}{3}$	$135°$ $\frac{3\pi}{4}$	$150°$ $\frac{5\pi}{6}$	$180°$ π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ej def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Mängdlära

$A \cap B = \{x \mid x \in A \text{ och } x \in B\}$ $\qquad$ $A \cup B = \{x \mid x \in A \text{ eller } x \in B\}$

$A \setminus B = \{x \mid x \in A \text{ och } x \notin B\}$ $\qquad$ $A^C = \{x \mid x \in G \text{ och } x \notin A\}$

Talteori

Kongruens $\quad a \equiv b (\text{mod } c)$ om differensen $a - b$ är delbar med c

Om $a_1 \equiv b_1 \ (\text{mod}\, c)$ och $a_2 \equiv b_2 \ (\text{mod}\, c)$ gäller att

1. $a_1 + a_2 \equiv b_1 + b_2 \ (\text{mod}\, c)$
2. $a_1 \cdot a_2 \equiv b_1 \cdot b_2 \ (\text{mod}\, c)$

Om $a \equiv b \ (\text{mod } c)$ gäller att

3. $m \cdot a \equiv m \cdot b \ (\text{mod } c)$ för alla heltal m
4. $a^n \equiv b^n \ (\text{mod}\, c)$ för alla heltal $n \geq 0$

Aritmetisk summa $\quad s_n = n \cdot \dfrac{a_1 + a_n}{2}$ där $a_n = a_1 + (n-1) \cdot d$

Geometrisk summa $\quad s_n = a_1 \dfrac{k^n - 1}{k - 1}$ där $a_n = a_1 \cdot k^{n-1}$

Kombinatorik

Permutationer $\quad P(n,k) = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1) = \dfrac{n!}{(n-k)!}$ där $0 \leq k \leq n$

Kombinationer $\quad C(n,k) = \dbinom{n}{k} = \dfrac{P(n,k)}{k!} = \dfrac{n!}{k!(n-k)!}$ där $0 \leq k \leq n$